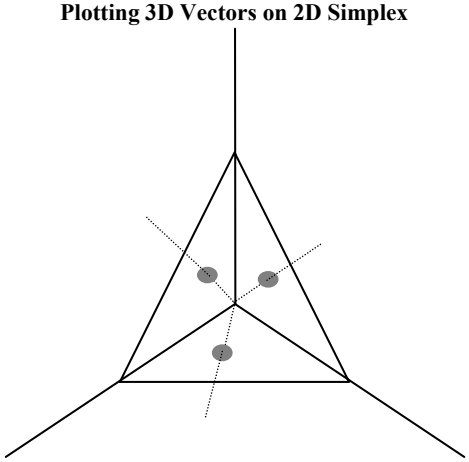
Peter K. Schott
UCLA
May 1998

How to Plot an Endowment Triangle

1 Intuition and Algebra

As illustrated in figure 1, the problem is to represent three dimensional vectors as points on a two dimensional simplex.

Figure 1
Plotting 3D Vectors on 2D Simplex

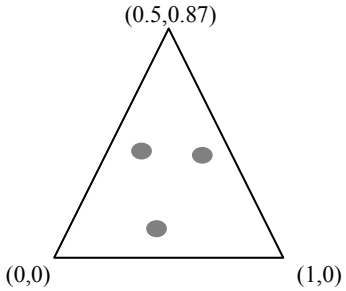


Thus, we need a 2x3 matrix ***A*** such that

$$\boldsymbol{y}=\boldsymbol{A}\boldsymbol{x},$$

where ***x*** is the 3x1 vector we start with and ***y*** is the 2x1 vector we wish to find. If we place the lower left hand vertex of the unit simplex at the origin, the boundaries of the simplex in two-space are as illustrated in figure 2.

Figure 2
Boundaries of the Unit Simplex in 2-Space



Then our problem is to find the ***A*** matrix which results in the following transformations:

$$(1,0,0) = (0,0)$$
$$(0,1,0) = (1,0)$$
$$(0,0,1) = (0.5,0.87)$$

A little manipulation reveals that

$$A = \begin{bmatrix} 0 & 0.50 & 1 \\ 0 & 0.87 & 0 \end{bmatrix}.$$

Often, tilting the simplex will allow for a more revealing display of points. To "tilt" the two dimensional representation, we need to normalize the three dimensional vector before translating it to two dimensions. If ***t*** is the 3x1 tilting vector, then

$$\boldsymbol{y=Ac},$$

where

$$c_{ij} = x_{ij} t_{ij} (x't)^{-1}.$$

2 Matlab Code

Transforming a matrix of three dimensional endowment vectors into an endowment triangle via matlab is quite easy. The following Matlab function `leamer.m` inputs a data matrix *x3* and produces the two dimensional co-ordinates *y2* that have been tilted according to the values in the vector *tilt*. The values in *y2* can then be plotted in 2 space via the `plot.m` routine.

```
[Y2] = function leamer(x3)
%Variable Definitions
%
%     x3       (x1,x2,x3) vector of raw endowments
%     tilt     simplex perspective vector (default=(1,1,1))
%     s3       tilt-scaled endowments
%     length   sum of tilt-scaled endowments
%     n3       normalized endowments=s/l
%     y2       (y1,y2) two dimensional coordinates

%Definintions
a      = [0 0.5 1; 0 sqrt(3/4) 0]';
tilt   = [1 1 1];

%Calculations
s3     = x3.*repmat(tilt,size(x3,1),1);
length = sum(s3,2);
n3     = s3./repmat(length,1,3);
y2     = n3*a;
```

3 References

Leamer, E.E.. 1987. Paths of development in the three-factor, *n*-good general equilibrium model. *Journal of Political Economy* 95, 961-999.

McKenzie, Lionel W. 1955. Equality of Factor Prices in World Trade. *Econometrica* 23(3):239-257.

Schott, Peter K. 1998. Education, Development and Trade: What's the Relationship? Mimeo.